Math 10A HW12

(1) True by definition!

(2) True by definition!

(3)

(a) $A-B=\begin{pmatrix}-1 & 2 & 0\\ 3 & 0 & 1\\ 2 & 1 & -1\end{pmatrix}-\begin{pmatrix}0 & 2 & 0\\ 0 & 1 & 1\\ -1 & -2 & 1\end{pmatrix}=\begin{pmatrix}-1 & 0 & 0\\ 3 & -1 & 0\\ 3 & 3 & -2\end{pmatrix}$

$A+B=\begin{pmatrix}-1 & 2 & 0\\ 3 & 0 & 1\\ 2 & 1 & -1\end{pmatrix}+\begin{pmatrix}0 & 2 & 0\\ 0 & 1 & 1\\ -1 & -2 & 1\end{pmatrix}=\begin{pmatrix}-1 & 4 & 0\\ 3 & 1 & 2\\ 1 & -1 & 0\end{pmatrix}$

$B-D$ undefined (different dimensions)

$\vec{r}-\vec{w}$ undefined (different dimensions)

(b) $A^T=\begin{pmatrix}-1 & 3 & 2\\ 2 & 0 & 1\\ 0 & 1 & -1\end{pmatrix}$ $C^T=\begin{pmatrix}3 & 0 & 1\\ -1 & 2 & -1\end{pmatrix}$

$D^T=\begin{pmatrix}4 & -1\\ 0 & 2\end{pmatrix}$, $\vec{v}^T=\begin{pmatrix}3\\ -1\end{pmatrix}$, $\vec{r}^T=\begin{pmatrix}4 & 2 & -7\end{pmatrix}$

(c) $2B = \begin{pmatrix} 0 & 4 & 0 \\ 0 & 2 & 2 \\ -2 & -4 & 2 \end{pmatrix}$ $\quad 7\vec{\omega} = (28 \;\; 14)$

$2\vec{u} + 3\vec{\omega}$ undefined ($\vec{u}, \vec{\omega}$ different dimensions)

$-p = \begin{pmatrix} -4 & 0 \\ 1 & -2 \end{pmatrix}$

(d) $\vec{v} \cdot \vec{\omega} = (3)(4) + (-1)(2) = 12 - 2 = 10$

$|\vec{v}| = \sqrt{9+1} = \sqrt{10}$

$|\vec{\omega}| = \sqrt{16+4} = \sqrt{20}$

$|\vec{u}| = \sqrt{9+1+25} = \sqrt{35}$

$|\vec{r}| = \sqrt{16+4+49} = \sqrt{69}$

$\vec{r} \cdot \vec{u}$ undefined (different dimensions)

$\vec{v} \cdot \vec{u}$ undefined (different dimensions)

(e) angle b/t $\vec{v}, \vec{\omega} \Rightarrow \cos\theta = \dfrac{\vec{v} \cdot \vec{\omega}}{|\vec{v}|\,|\vec{\omega}|} = \dfrac{10}{\sqrt{10} \cdot \sqrt{20}}$

$\cos\theta = 1/\sqrt{2} \rightarrow \theta = 45°$

angle b/t $\vec{r}, \vec{u} \Rightarrow \vec{r} \cdot \vec{u}$ undefined (part d)

(f) $\vec{v}\vec{\omega}$ undefined $(1 \times \overbrace{2)(1}^{\neq} \times 2)$

$\vec{\omega}\vec{v}$ undefined $(1 \times \overbrace{2)(1}^{\neq} \times 2)$

$\vec{v}\vec{u}$ undefined $(1 \times \overbrace{2)(1}^{\neq} \times 3)$

$\vec{r}\vec{u} = \underset{3\times 1}{\begin{pmatrix} 4 \\ 2 \\ -7 \end{pmatrix}} \underset{1\times 3}{(3 \;\; -1 \;\; 5)} = \begin{pmatrix} 12 & -4 & 20 \\ 6 & -2 & 10 \\ -21 & 7 & -35 \end{pmatrix}$

(g) $A\vec{r} = \begin{pmatrix} -1 & 2 & 0 \\ 3 & 0 & 1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ -7 \end{pmatrix} = \begin{pmatrix} -4+4+0 \\ 12+0-7 \\ 8+2+7 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 17 \end{pmatrix}$

$C\vec{v}$ not defined $(3\times \underbrace{2)(1}_{\neq}\times 2)$

$D\vec{r}$ not defined $(2\times \underbrace{2)(3}_{\neq}\times 1)$

$B\vec{r} = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ -7 \end{pmatrix} = \begin{pmatrix} 4 \\ 2-7 \\ -4-4-7 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ -15 \end{pmatrix}$

(h) $CD = \begin{pmatrix} 3 & -1 \\ 0 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 12+1 & -2 \\ -2 & 4 \\ 4+1 & -2 \end{pmatrix}$

DC not defined $(2\times \underbrace{2)(3}_{\neq}\times 2)$

$AB = \begin{pmatrix} -1 & 2 & 0 \\ 3 & 0 & 1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ -1 & 4 & 1 \\ 1 & 7 & 0 \end{pmatrix}$

(4) False, can only find inverse for $n \times n$ matrix

(5) True, $\det(A) \neq 0$ unique solt'n

(6) True, ALWAYS have $\vec{x} = \vec{0}$ solt'n

(7)

(a) $\det\begin{pmatrix} -1 & 2 & 0 \\ 3 & 0 & 1 \\ 2 & 1 & -1 \end{pmatrix} = -1\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix}$

$= -1(0-1) - 2(-3-2) = -1(-1) - 2(-5)$

$\det A = 1 + 10 = 11$

$\det B = \det\begin{pmatrix} 0 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & 1 \end{pmatrix} = -2\begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} = -2(0+1) = -2$

$\det C = \det\begin{pmatrix} 0 & 2 & 3 \\ -1 & 1 & 5 \\ 3 & -1 & -12 \end{pmatrix} = -2\begin{vmatrix} -1 & 5 \\ 3 & -12 \end{vmatrix} + 3\begin{vmatrix} -1 & 1 \\ 3 & -1 \end{vmatrix}$

$= -2(12-15) + 3(1-3) = -2(-3) + 3(-2)$

$= 6 - 6 = 0$

$\det D = -8 - (-9) = 1$

$\det E = 8 - 0 = 8$

$\det F = -18 - (-18) = 0$

(b) $D^{-1} = \frac{1}{\det D}\begin{pmatrix} -4 & -3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -4 & -3 \\ 3 & 2 \end{pmatrix}$

$$E^{-1} = \frac{1}{\det E}\begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix}$$

F^{-1} not defined since $\det(F) = 0$

(c) $\begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$

$$\Rightarrow \begin{cases} 4x_1 + 0x_2 = 4 \\ -x_1 + 2x_2 = -4 \end{cases}$$

(d) $A\vec{x} = \vec{b}_2$ $\quad \det(A) \neq 0 \Rightarrow$ unique solt'n
$B\vec{x} = \vec{b}_2$ $\quad \det(B) \neq 0 \Rightarrow$ unique solt'n
$C\vec{x} = \vec{b}_2$ $\quad \det(C) = 0 \Rightarrow$ NOT unique solt'n
$D\vec{x} = \vec{b}_1$ $\quad \det(D) \neq 0 \Rightarrow$ unique solt'n
$E\vec{x} = \vec{b}_1$ $\quad \det(E) \neq 0 \Rightarrow$ unique solt'n
$F\vec{x} = \vec{b}_1$ $\quad \det(F) = 0 \Rightarrow$ NOT unique solt'n

(8) $\begin{vmatrix} 4 & -3 & 2 \\ 1 & -3 & 1 \\ 9 & 1 & 0 \end{vmatrix} = 9\begin{vmatrix} -3 & 2 \\ -3 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix}$

$= 9(-3+6) - (4-2) = 9(3) - 2 = 34$

$\begin{vmatrix} 6 & 7 & -2 \\ -2 & -3 & 1 \\ 7 & 7 & 1 \end{vmatrix} = 6\begin{vmatrix} -3 & 1 \\ 7 & 1 \end{vmatrix} + 2\begin{vmatrix} 7 & -2 \\ 7 & 1 \end{vmatrix} + 7\begin{vmatrix} 7 & -2 \\ -3 & 1 \end{vmatrix}$

$= 6(-3-7) + 2(7+14) + 7(7-6)$

$= 6(-10) + 2(21) + 7 = -60 + 42 + 7$

(9) True, a row of the form $(0\ 0 \dots 0 \mid 1)$ corresponds to an equation $0x_1 + \dots + 0x_n = 1$

(10) False, see class worksheet

(11) True!

(12)

(a) $\left(\begin{array}{ccc|c} 2 & 1 & 8 & 1 \\ -1 & 1 & -1 & 0 \\ -2 & 5 & 4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} +1 & -1 & +1 & 0 \\ 2 & 1 & 8 & 1 \\ -2 & 5 & 4 & 1 \end{array}\right) \begin{array}{l} R_1 \leftrightarrow R_2 \\ -R_1 \\ \end{array}$

$\sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & 6 & 1 \\ 0 & 3 & 6 & 0 \end{array}\right) \begin{array}{l} \\ -2R_1 + R_2 \\ 2R_1 + R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 3 & 6 & 1 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 1/3 \end{array}\right) \begin{array}{l} +R_2 + R_1 \\ \\ \frac{1}{3} R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1/3 \end{array}\right) \begin{array}{l} \\ \\ -R_2 + R_3 \end{array}$

(b) $\left(\begin{array}{ccc|c} 4 & 5 & 0 & 2 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 4 & 5 & 0 & 2 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & -3 & -4 & -2 \end{array}\right) \begin{array}{l} \\ \\ -4R_1 + R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & -5 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 5 & -2 \end{array}\right) \begin{array}{l} -2R_2 + R_1 \\ \\ 3R_2 + R_3 \end{array}$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -5 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & -2/5 \end{array}\right) \begin{array}{l} \\ \\ (\tfrac{1}{6})R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 6/5 \\ 0 & 0 & 1 & -2/5 \end{array}\right) \begin{array}{l} 5R_3+R_1 \\ -3R_3 \\ +R_2 \end{array}$$

(c) $$\left(\begin{array}{ccc|c} 3 & 2 & 5 & 2 \\ 2 & 0 & 1 & 5 \\ 0 & 0 & 4 & -4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 1 & 5 \\ 3 & 2 & 5 & 2 \\ 0 & 0 & 4 & -4 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 6 & 0 & 3 & 15 \\ 6 & 12 & 10 & 10 \\ 0 & 0 & 1 & -1 \end{array}\right) \begin{array}{l} 3R_1 \\ 2R_2 \\ \frac{1}{4}R_3 \end{array} \sim \left(\begin{array}{ccc|c} 6 & 0 & 3 & 15 \\ 0 & 12 & 7 & -5 \\ 0 & 0 & 1 & -1 \end{array}\right) -R_1+R_2$$

$$\sim \left(\begin{array}{ccc|c} 6 & 0 & 0 & 18 \\ 0 & 12 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right) \begin{array}{l} -3R_3+R_1 \\ -7R_3+R_2 \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1/6 \\ 0 & 0 & 1 & -1 \end{array}\right) \begin{array}{l} \frac{1}{6}R_1 \\ \frac{1}{12}R_2 \\ \end{array}$$

(13) $$\left(\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 1 & 2 & 1 & 3 \\ 1 & 3 & 3 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & -1 & 0 \\ 1 & 3 & 3 & 7 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & -3 & -6 \\ 0 & 1 & 2 & 4 \end{array}\right) \begin{array}{l} \\ -2R_1+R_2 \\ -R_1+R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & -1 & -3 & -6 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -3 & -5 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & -2 \end{array}\right) \begin{array}{l} -2R_2+R_1 \\ \\ R_2+R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & -3 & -5 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -3 & -5 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right) \begin{array}{l} 3R_3+R_1 \\ -2R_3+R_2 \\ \end{array}$$

$\Rightarrow$ $x=1$

$y=0$

$z=2$

(14) $\begin{pmatrix} 1 & 2 & -1 \\ -3 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix}^{-1}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -3 & 1 & 2 & 0 & 1 & 0 \\ -2 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \underset{2R_1+R_3}{\overset{3R_1+R_2}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 7 & -1 & 3 & 1 & 0 \\ 0 & 6 & -1 & 2 & 0 & 1 \end{array}\right)$$

$$\begin{array}{c} \sim \\ \frac{1}{7}R_2 \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1/7 & 3/7 & 1/7 & 0 \\ 0 & 6 & -1 & 2 & 0 & 1 \end{array}\right) \begin{array}{c} \sim \\ -6R_2 \\ +R_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1/7 & 3/7 & 1/7 & 0 \\ 0 & 0 & -1/7 & -4/7 & -6/7 & 1 \end{array}\right)$$

$$\begin{array}{c} -2R_2+R_1 \\ \sim \\ \\ -7R_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & -5/7 & 1/7 & -2/7 & 0 \\ 0 & 1 & -1/7 & 3/7 & 1/7 & 0 \\ 0 & 0 & 1 & 4 & 6 & -1/7 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -5/7 & 1/7 & -\frac{2}{7} & 0 \\ 0 & 1 & -1/7 & 3/7 & 1/7 & 0 \\ 0 & 0 & 1/7 & 4/7 & 6/7 & -1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & +4 & -5 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1/7 & 4/7 & 6/7 & -1 \end{array}\right) \begin{array}{l} 5R_3+R_1 \\ R_3+R_2 \\ \end{array}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & +4 & -5 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 4 & 6 & -7 \end{array}\right) \; 7R_3$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & -1 \\ -3 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & +4 & -5 \\ 1 & 1 & -1 \\ 4 & 6 & -7 \end{pmatrix}$$

Check $\begin{pmatrix} 1 & 2 & -1 \\ -3 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & +4 & -5 \\ 1 & 1 & -1 \\ 4 & 6 & -7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ✓